

## Do Matching Frictions Explain Unemployment? Not in Bad Times<sup>†</sup>

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The long queues of unemployed workers at job bureaus and factory gates observed during the Great Depression suggest that jobs are lacking in recessions, irrespective of frictions in matching unemployed workers to recruiting firms. Existing search-and-matching models of unemployment, either with bargained wages as in Pissarides (2000) or with rigid wages as in Hall (2005a), converge asymptotically to full employment when matching frictions disappear, which makes these models inadequate to study recessionary unemployment. In contrast, this paper proposes a search-and-matching model in which jobs are rationed in recessions: the labor market does not clear at the limit where matching frictions are absent. By constructing a model in which job rationing arises in equilibrium in a frictional labor market, one can begin to understand its macroeconomic implications more rigorously.

The distinctive feature of the model is that in recessions, jobs are rationed in the sense that some unemployment remains in the absence of matching frictions. *Rationing unemployment* measures the shortage of jobs in the absence of matching frictions, and *frictional unemployment* measures additional unemployment attributable to matching frictions. In existing models, all unemployment is frictional at any point of the business cycle. In expansions, all unemployment is also frictional in my model. The fundamental property of my model, and its point of departure from existing models, is that in recessions rationing unemployment increases and drives the rise in total unemployment, while frictional unemployment decreases. A related property is that, instead of remaining constant over the business cycle as in existing models, the positive effect of a marginal reduction in matching frictions on unemployment decreases sharply in recessions. The macroeconomic implication is that, even though the labor market always sees vast flows of jobs and workers and a great deal of matching activity, recessions are periods of acute job shortage during which matching frictions have little influence on labor market outcomes.

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To embed job rationing in a labor market with matching frictions, this paper builds on the Pissarides (2000) model by relaxing two key assumptions: completely flexible wages and constant marginal returns to labor, which impose a horizontal aggregate demand for labor. These assumptions are critical because *either* implies that the economy would be at full employment absent matching frictions.<sup>1</sup> Specifically, large firms post vacancies to hire new workers in response to exogenous job destruction and technology shocks. Recruiting is costly because of matching frictions. It is especially so in expansions, when the labor market is tight as firms post many vacancies filled from a small pool of unemployed workers. Firms face diminishing marginal returns to labor; therefore the aggregate demand for labor is downward-sloping. Wages are rigid as they do not adjust as much as technology. Wages, however, respect the private efficiency of worker-firm matches because they remain in the interval between the flow value of unemployment and the marginal product of labor, and do not cause the inefficient destruction of worker-firm matches, generating a positive bilateral surplus.

The mechanism yielding job rationing is quite simple. Absent recruiting expenses, firms hire workers until marginal product of labor equals wage. After a negative technology shock, the marginal product of labor falls but rigid wages adjust downward only partially. The marginal product of labor decreases with employment by diminishing marginal returns. If the adverse shock is sufficiently large, the marginal product of the least productive workers falls below the wage. It becomes unprofitable for firms to hire these workers. Some unemployment, which I call rationing unemployment, remains even if recruiting cost is zero. With a positive recruiting cost, the marginal cost of labor is higher and firms reduce employment. The resulting amount of additional unemployment is frictional unemployment. In recessions, technology falls further, rationing unemployment increases, driving the rise in total unemployment. Many unemployed workers apply to the few vacancies left. It becomes easier for firms to recruit. Each vacant job is filled rapidly and at low cost in spite of matching frictions. The contribution of recruiting expenses to the marginal cost of labor falls, and frictional unemployment falls.

To illustrate these theoretical findings, I calibrate the model with US data. I simulate the impact of technology shocks and find that simulated moments for labor market variables are close to their empirical counterparts. Critically, even a low estimate of wage rigidity, such as that obtained by Haefke, Sonntag, and Van Rens (2008) using micro-data reporting earnings of new hires, is sufficient for the model to amplify shocks as much as in the data. Then, I construct a historical time series for unemployment by simulating the model with the technology series measured in US data and decompose this series, which matches actual unemployment closely, into historical time series for frictional and rationing unemployment. The decomposition uncovers large cyclical fluctuations in frictional and rationing unemployment. In the model, as long as unemployment is below 4.8 percent, it is all frictional. On average, unemployment amounts to 5.8 percent of the labor force, frictional unemployment to

<sup>1</sup>Section II proves that all workers are employed when matching frictions disappear in the canonical search-and-matching model (Pissarides 2000; Shimer 2005; Hagedorn and Manovskii 2008), its variant with rigid wages (Shimer 2004; Hall 2005a; Blanchard and Galí 2010; Gertler and Trigari 2009), or its variant with diminishing marginal returns to labor (Elsby and Michaels 2008; Cahuc, Marque, and Wasmer 2008).

3.7 percent, and rationing unemployment to 2.1 percent. In recessions, when unemployment reaches 8.0 percent, rationing unemployment increases above 6.0 percent, while frictional unemployment decreases below 2.0 percent.

Empirical observations suggest that matching frictions and job rationing are important sources of unemployment.<sup>2</sup> First, labor markets see constant job destruction, job creation, and large flows of workers (Blanchard and Diamond 1989; Davis, Haltiwanger, and Schuh 1996). Accordingly, frictions hindering matching of workers and firms create unemployment.<sup>3</sup> Second, ethnographic and historical works document barriers to the short-run adjustment of wages to productivity or labor market conditions: (i) the widespread organization of firms around internal labor markets, motivated by concerns for equity, which tie wages to job description (Doeringer and Piore 1971; Jacoby 1984; Baker, Gibbs, and Holmstrom 1994); (ii) labor market institutions (Temin 1990; Gorodnichenko, Mendoza, and Tesar 2012); and (iii) the managerial best practice of avoiding pay cuts, which antagonize workers and reduce profitability (Campbell and Kamlani 1997; Bewley 1999; Krueger and Mas 2004; Mas 2006). These barriers constrain wages to remain above market-clearing level irrespective of matching frictions, leading to a shortage of jobs.<sup>4</sup>

The paper is organized as follows. Section I presents the search-and-matching framework. Section II shows that existing search-and-matching models do not have job rationing. Section III shows that job rationing arises from the combination of wage rigidity and diminishing marginal returns to labor. It proves that total and rationing unemployment increase in recessions, whereas frictional unemployment decreases. It also proves that the elasticity of unemployment with respect to recruiting cost falls in recessions. Section IV calibrates the model to quantify fluctuations of unemployment and its components. To conclude, Section V discusses applications of these results to labor market policies. Derivations, proofs, and robustness checks are collected in a separate online Appendix.

## I. General Model

This is a discrete-time model. Fluctuations are driven by technology, which follows a Markov process  $\{a_t\}_{t=0}^{+\infty}$ .<sup>5</sup> There is a unit mass of workers in the labor market, either employed, or unemployed and searching for a job. Workers have risk-neutral

<sup>2</sup>The literature on unemployment is vast. Alternative models include mismatch models (Shimer 2007), wage-posting models (Burdett and Mortensen 1998), models in which jobseekers exert costly search efforts but frictions prevent them from all finding a job (Christiano, Trabandt, and Walentin 2010).

<sup>3</sup>To model matching frictions, I follow the literature and impose a vacancy-posting cost (Pissarides 1985; Mortensen and Pissarides 1994; Pissarides 2000; Shimer 2005; Hall 2005a). Other papers impose a hiring cost instead (Gertler and Trigari 2009; Gertler, Sala, and Trigari 2008). Empirical studies show that hiring costs also influence firms' recruiting behavior (Yashiv 2000; Carlsson, Eriksson, and Gottfries 2006; Christiano, Trabandt, and Walentin forthcoming).

<sup>4</sup>Models of job rationing include efficiency-wage models (Solow 1979), gift-exchange models (Akerlof 1982), insider-outsider models (Lindbeck and Snower 1988), social-norm models (Akerlof 1980), or shirking models (Alexopoulos 2004, 2007). Recent works by Galí (2011) and Galí, Smets, and Wouters (forthcoming) introduce job rationing into general-equilibrium models by assuming that wages are at a markup above labor supply.

<sup>5</sup>Empirical evidence suggests that recessions are driven by aggregate-activity shocks and not by reallocation shocks (Abraham and Katz 1986; Blanchard and Diamond 1989). In line with the literature, I assume a stable matching function and introduce aggregate technology shocks.

preferences over consumption, and discount future payoffs by a factor  $\delta \in (0, 1)$ .<sup>6</sup> Each period, workers consume all their income (wage if employed, nothing if unemployed).

### A. Labor Market

A continuum of firms indexed by  $i \in [0, 1]$  hire workers. At the end of period  $t - 1$ , a fraction  $s$  of the  $n_{t-1}$  existing worker-job matches are exogenously destroyed.<sup>7</sup> Workers who lose their job can apply for a new job immediately. At the beginning of period  $t$ ,  $u_t$  unemployed workers are looking for a job:

$$(1) \quad u_t = 1 - (1 - s) \cdot n_{t-1}.$$

Firms open  $v_t$  vacancies to recruit unemployed workers. The number of matches made in period  $t$  is given by a constant-returns matching function  $h(u_t, v_t)$ , differentiable and increasing in both arguments, with the restriction that  $h(u_t, v_t) \leq \min(u_t, v_t)$ . Conditions on the labor market are summarized by the labor market tightness  $\theta_t \equiv v_t/u_t$ . An unemployed worker finds a job with probability  $f(\theta_t) \equiv h(u_t, v_t)/u_t = h(1, \theta_t)$ , and a vacancy is filled with probability  $q(\theta_t) \equiv h(u_t, v_t)/v_t = h(1/\theta_t, 1)$ .<sup>8</sup> In a tight market it is easy for job seekers to find jobs—the job-finding probability  $f(\theta_t)$  is high—and difficult for firms to hire—the job-filling probability  $q(\theta_t)$  is low. Keeping a vacancy open has a per-period cost  $c \cdot a_t$ .<sup>9</sup> The recruiting cost  $c \in (0, +\infty)$  captures the resources that firms must spend to recruit workers because of matching frictions. I assume no randomness at the firm level: firm  $i$  hires  $h_t(i) \geq 0$  workers with certainty by opening  $h_t(i)/q(\theta_t)$  vacancies and spending  $[c \cdot a_t/q(\theta_t)]h_t(i)$ . When the labor market is tighter, a vacancy is less likely to be filled, a firm posts more vacancies to fill a job, and recruiting is more costly. Aggregate number of hires  $h_t = \int_0^1 h_t(i) di$ , labor market tightness  $\theta_t$ , and unemployment  $u_t$  are related through the job-finding probability

$$(2) \quad f(\theta_t) = \frac{h_t}{u_t}.$$

<sup>6</sup>There is no labor-supply decision, neither number of hours nor labor force participation. This setup is standard (Mortensen and Pissarides 1994; Pissarides 2000; Shimer 2005; Hall 2005a; Hall and Milgrom 2008; Gertler and Trigari 2009). It is consistent with empirical evidence: (i) most cyclical variation in total hours worked is due to variation in number of employed workers, and not variation in hours per worker; and (ii) most cyclical variation in unemployment is due to variation in number of employed workers, and not variation in number of labor-force participants (Shimer 2010, Chapter 1).

<sup>7</sup>Recent evidence suggests that job destructions may be countercyclical (Elsby, Michaels, and Solon 2009; Fujita and Ramey 2009). The model ignores this issue, and assumes a constant job destruction rate. Understanding job destruction and its interaction with job rationing is an important task for future research. Barnichon (2010a) is a first attempt at modeling endogenous job destructions when firms adjust employment and hours in response to demand shocks.

<sup>8</sup>Due to the restriction  $h(u_t, v_t) \leq \min(u_t, v_t)$ , the probabilities satisfy  $f(\theta_t) \leq 1$  and  $q(\theta_t) \leq 1$ .

<sup>9</sup>As in Pissarides (2000), the per-period cost of opening a vacancy  $c \cdot a$  is linear in technology  $a$ . This assumption can be justified on the grounds that recruiting is a labor-intensive activity, so the per-period cost to maintain a vacancy open depends on wages, which are either proportional (with wage bargaining) or nearly proportional (with rigid wages,  $w_t \propto a_t^\gamma$  with  $\gamma = 0.7$ ) to technology. This assumption simplifies derivations and exposition. The online Appendix shows it has virtually no effect on the results.

In steady state, inflows to unemployment  $s \cdot n$  equals outflows from unemployment  $[1 - (1 - s)n] \cdot f(\theta)$ , and labor market tightness  $\theta$  is related to employment  $n$  by the Beveridge curve

$$(3) \quad n = \frac{1}{(1 - s) + s/f(\theta)}.$$

If firms post more vacancies  $v$ , labor market tightness  $\theta$  increases, which raises the probability  $f(\theta)$  to find a job and increases employment  $n$ . As in Blanchard and Galí (2010), I assume that the  $h_t$  newly hired workers become productive immediately upon hiring at the beginning of period  $t$ , and participate in production during period  $t$  with the  $(1 - s) \cdot n_{t-1}$  incumbent workers.

### B. Wage Schedule

The wage is set once worker and firm have matched. The marginal product of labor always exceeds the flow value of unemployment, normalized to zero, so there are always mutual gains from matching. There is no compelling theory of wage determination in such an environment (Shimer 2005; Hall 2005a). Hence, I specify a general wage schedule that does not result from a particular wage-setting mechanism but nests as special cases the outcomes of a broad set of mechanisms: generalized Nash bargaining, Stole and Zwiebel (1996) intrafirm bargaining, and various reduced-form rigid wages. Let

$$(4) \quad w_t(i) = w(n_t(i), \theta_t, n_t, a_t),$$

where  $n_t(i)$  and  $w_t(i)$  are number of workers and wage paid in firm  $i$  at time  $t$ . The function  $w$  is continuous and differentiable in all arguments. The wage  $w_t(i)$  is affected by various factors: technology  $a_t$  and employment  $n_t(i)$ , which determine current marginal productivity in firm  $i$ ; labor market tightness  $\theta_t$ , which determines outside opportunities of firms and workers; and the state of the economy  $(n_t, a_t)$ , as expectations about future economic outcomes conditional on time-  $t$  information are measurable with respect to  $(n_t, a_t)$ .<sup>10</sup>

### C. Firms

Firm  $i$  takes price as given and ranks profit streams according to

$$(5) \quad \mathbb{E}_0 \sum_{t=0}^{+\infty} \delta^t \cdot \pi_t(i),$$

<sup>10</sup>In a symmetric environment, aggregate employment  $n_t$  and technology  $a_t$  summarize the information set at time  $t$  because technology follows a Markov process.

where  $\mathbb{E}_0$  denotes the mathematical expectation conditioned on time-0 information and  $\pi_t(i)$  is the real profit of firm  $i$  in period  $t$ :

$$\pi_t(i) = g(n_t(i), a_t) - w_t(i) \cdot n_t(i) - \frac{c \cdot a_t}{q(\theta_t)} \cdot h_t(i).$$

The production function  $g$  is differentiable and increasing in both arguments. The firm faces a constraint on the number of workers employed each period:

$$(6) \quad n_t(i) \leq (1 - s) \cdot n_{t-1}(i) + h_t(i).$$

**DEFINITION 1:** *Taking as given the wage schedule (4), as well as stochastic processes for labor market tightness, aggregate employment, and technology  $\{\theta_t, n_t, a_t\}_{t=0}^{+\infty}$ , the firm's problem is to choose stochastic processes  $\{h_t(i), n_t(i)\}_{t=0}^{+\infty}$  to maximize (5) subject to the sequence of recruitment constraints (6). The time- $t$  element of a firm's choice must be measurable with respect to  $(a^t, n_{-1})$ , where  $a^t \equiv (a_0, a_1, \dots, a_t)$ .*

In equilibrium, firms do not close jobs beyond those destroyed for exogenous reasons, so equation (6) becomes  $h_t(i) = n_t(i) - (1 - s) \cdot n_{t-1}(i)$ . For any recruiting cost  $c \in (0, +\infty)$ , I assume that the maximization problem is concave with an interior solution. Thus, employment satisfies the first-order condition

$$(7) \quad \frac{\partial g}{\partial n(i)}(n_t(i), a_t) = w_t(i) + \frac{c \cdot a_t}{q(\theta_t)} + n_t(i) \cdot \frac{\partial w}{\partial n(i)}(n_t(i), \theta_t, n_t, a_t) - \delta(1 - s) \mathbb{E}_t \left[ \frac{c \cdot a_{t+1}}{q(\theta_{t+1})} \right].$$

This condition implies that firm  $i$  hires labor until marginal product of labor  $\partial g / \partial n$  equals the marginal cost of labor, which is the sum of wage  $w_t(i)$ , hiring cost  $c \cdot a_t / q(\theta_t)$ , change in the wage bill from marginally increasing employment  $n_t(i) \cdot \partial w / \partial n(i)$ , minus discounted cost of hiring next period  $\delta(1 - s) \mathbb{E}_t [c \cdot a_{t+1} / q(\theta_{t+1})]$ .

#### D. Equilibrium

**DEFINITION 2:** *The wage process  $\{w_t(i)\}_{t=0}^{+\infty}$  is privately efficient if all worker-firm  $i$  pairs exploit all opportunities for mutual improvement. Let  $n_t^* \equiv (1 - s) \cdot n_{t-1}$ . If firms are symmetric, a necessary and sufficient condition for private efficiency is*

$$(8) \quad 0 < w_t < \frac{\partial g}{\partial n(i)}(n_t^*, a_t) - n_t^* \cdot \frac{\partial w}{\partial n(i)}(n_t^*, 0, n_t^*, a_t) + \delta(1 - s) \mathbb{E}_t \left[ \frac{c \cdot a_{t+1}}{q(\theta_{t+1})} \right].$$

Condition (8) implies that private efficiency is respected if wages remain low enough.<sup>11</sup> Private efficiency guarantees that wage rigidity never causes the destruction of a match generating a positive bilateral surplus, a reasonable equilibrium requirement when rational workers and firms engage in long-term interactions (Barro 1977; Hall 2005a).

**DEFINITION 3:** *Given initial employment  $n_{-1}$  and a stochastic process  $\{a_t\}_{t=0}^{+\infty}$  for technology, a symmetric equilibrium is a collection of stochastic processes  $\{n_t, h_t, \theta_t, u_t, w_t\}_{t=0}^{+\infty}$  that solve the firm's problem, satisfy the law of motion for unemployment (1), the law of motion for labor market tightness (2), the wage schedule (4), and the condition for private efficiency of all worker-firm pairs (8).*

In equilibrium, neither worker nor firm would choose to break an existing match since any match generates some surplus.

## II. Absence of Job Rationing in Existing Search-and-Matching Models

This section chooses production functions and wage schedules to specialize the general framework of Section I to three influential search-and-matching models: the canonical search-and-matching model, its variant with diminishing marginal returns to labor, and its variant with wage rigidity. I demonstrate that jobs are not rationed in these models: the economy converges to full employment when firms do not need to devote any resources, time or material, to recruiting. The analysis in this section and the next focuses on a static environment without aggregate shocks and with a labor market in steady state ( $a_t = a$  for all  $t$  and equation (3) holds) to be able to study the equilibrium theoretically and represent it diagrammatically; besides, the analysis delivers the same qualitative predictions as the study of a stochastic environment.<sup>12</sup>

### A. Canonical Model

ASSUMPTION 1:  $g(n, a) = a \cdot n$ .

ASSUMPTION 2: *There exists  $\beta \in (0, 1)$  such that*

$$(9) \quad w(n_t(i), \theta_t, n_t, a_t) = \frac{c\beta}{1 - \beta} \left\{ \frac{a_t}{q(\theta_t)} + \delta(1 - s) \mathbb{E}_t \left[ a_{t+1} \left( \theta_{t+1} - \frac{1}{q(\theta_{t+1})} \right) \right] \right\}.$$

<sup>11</sup>I derive condition (8) in the online Appendix. Since the income received during unemployment is zero, workers prefer working for any positive wage than quitting to unemployment, so a worker would never break an employment relationship with  $0 < w_t$ . If the income received during unemployment were  $b \geq 0$  (from household production or unemployment benefits), the first inequality would be  $b < w_t$ .

<sup>12</sup>Shimer (2005) and Pissarides (2009) argue that the equilibrium in a static environment with technology  $a$  approximates well the equilibrium in a stochastic environment when the realization of technology is  $a_t = a$  for two reasons: (i) the labor market rapidly converges to an equilibrium in which inflows to and outflows from employment are balanced because rates of inflow to and outflow from unemployment are large (Hall 2005b; Shimer 2012); and (ii) technology is very persistent (it is quite autocorrelated and shocks are of small amplitude). The online Appendix validates this approximation with numerical simulations.

LEMMA 1: Assume that  $w_t(i)$  in any period  $t$  and firm  $i$  is determined by generalized Nash bargaining, and  $\beta \in (0, 1)$  is workers' bargaining power. Then  $w_t(i)$  satisfies equation (9).

Lemma 1 shows that wage schedule (9) is the generalized Nash bargaining solution, which allocates a fraction  $\beta$  of the match surplus to the worker, and the rest to the firm. The canonical model retains the key elements of the models studied in Pissarides (2000) and Shimer (2005). In a static environment, equilibrium labor market tightness is characterized by

$$(10) \quad (1 - \beta) = c \cdot \left[ \frac{1 - \delta \cdot (1 - s)}{q(\theta)} + \delta \cdot (1 - s) \cdot \beta \cdot \theta \right],$$

which is obtained by combining firm's optimality condition (7) with wage (9). Equations (3) and (10) uniquely define equilibrium employment and tightness as implicit functions  $n(c)$  and  $\theta(c)$  of recruiting cost  $c$ . Since the Nash bargained wage is proportional to technology  $a$ —as shown in equation (9)—employment is independent of technology and unemployment does not fluctuate over the business cycle. Furthermore, Proposition 1 shows that the economy converges to full employment when matching frictions vanish.<sup>13</sup>

PROPOSITION 1: Under Assumption 1 and Assumption 2,  $\lim_{c \rightarrow 0} \theta(c) = +\infty$  and  $\lim_{c \rightarrow 0} n(c) = 1$ .

A simple diagram in Figure 1 explains this result. Using equation (3), I express labor market tightness  $\theta$  as an increasing function of employment  $n$  and represent equation (10) on a plane with employment on the  $x$ -axis. The upward-sloping line is marginal recruiting expenses, imposed by the presence of a positive recruiting cost  $c$  directly through the opening of vacancies and indirectly through wage bargaining, on the right-hand side of equation (10).<sup>14</sup> The horizontal line is the marginal profit from hiring labor gross of recruiting expenses, here simply the marginal product of labor, on the left-hand side of equation (10). The intersection of these two curves determines equilibrium employment.

Consider a decrease in recruiting cost  $c$ . The amortized cost of recruiting  $[1 - \delta(1 - s)] \cdot c \cdot a/q(\theta)$  decreases. The firm's surplus from an established relationship is the hiring cost  $c \cdot a/q(\theta)$  because a firm can immediately replace a worker at that cost. With Nash bargaining when the firm's surplus falls, so does the

<sup>13</sup>Even though the setup is slightly different, equilibrium conditions are similar to those of Pissarides (2000, Chapter 1). The main difference is that, in Pissarides (2000), unemployed workers receive unemployment benefits  $b > 0$ , constant and independent of technology, whereas I assume  $b = 0$ . To accommodate  $b > 0$ , I would replace  $(1 - \beta)$  by  $(1 - \beta)(1 - b/a)$  in (10). This change leads negative technology shocks to reduce the surplus of matches because the outside option of unemployed workers becomes relatively more attractive. Hence, firms reduce the number of vacancies posted, increasing unemployment, but only slightly (Shimer 2005). In contrast, unemployment is completely invariant to technology in my model. This property is quite general, as it holds (i) if unemployed workers receive unemployment benefits proportional to the average wage rate:  $b = r \cdot w$ , where  $r$  is the replacement rate (Pissarides 2000, Chapter 1); or (ii) if unemployed workers enjoy utility from leisure using a standard utility specification (Blanchard and Galí 2010). On the other hand, introducing  $b > 0$  would not modify the critical finding of Proposition 1 that there is no job rationing in the canonical model (as long as  $b < a$ ).

<sup>14</sup>Equation (10) is normalized by  $a/(1 - \beta)$ . Similarly, I normalize equations (12), (13), and (14) by  $a$ .

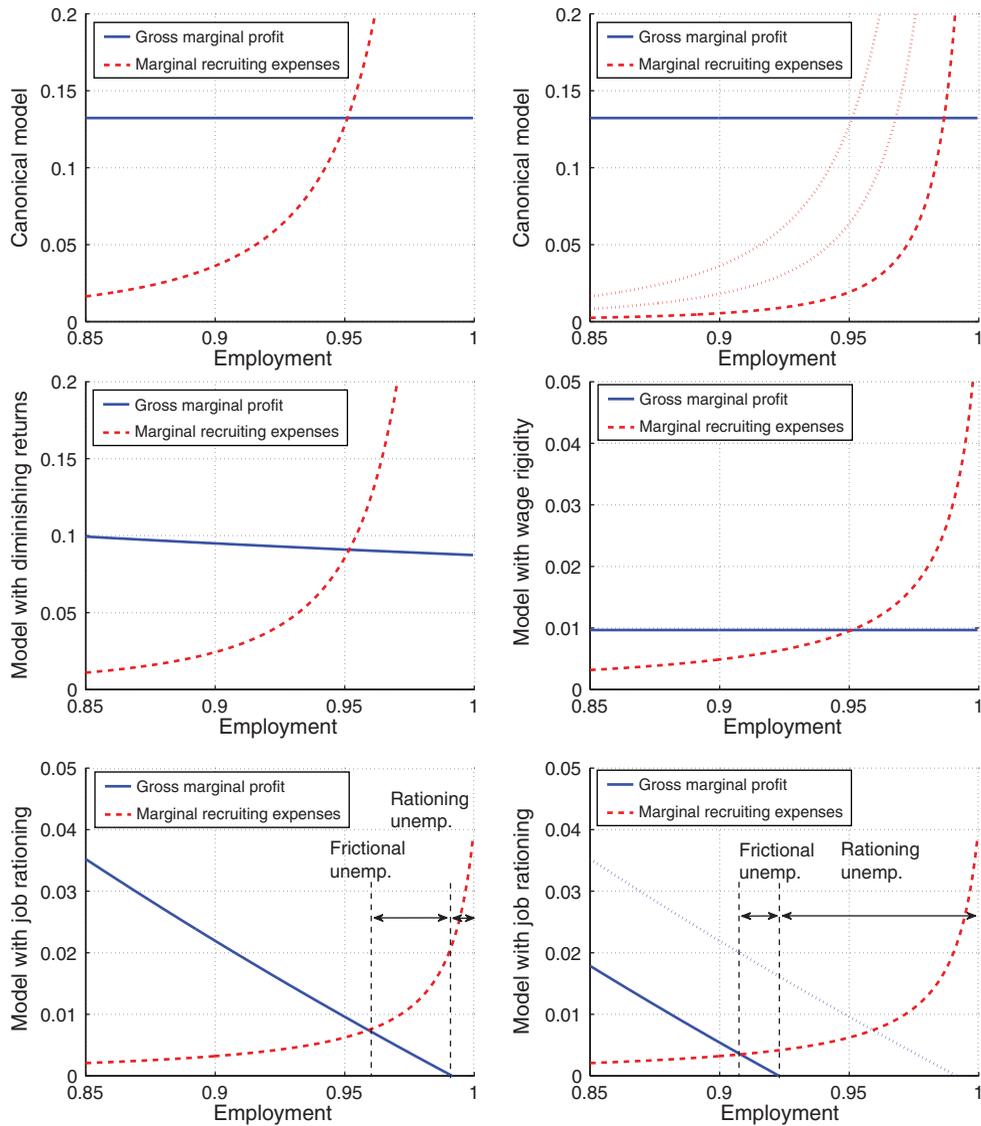


FIGURE 1. EQUILIBRIA IN VARIOUS SEARCH-AND-MATCHING MODELS

Notes: These diagrams describe equilibria in static environments in the models described in Section II and III. The two diagrams for the canonical model represent the equilibrium for a calibrated recruiting cost (left) and lower recruiting costs (right). The two diagrams for the model with job rationing represent the decomposition of unemployment into rationing and frictional unemployment for high technology (left) and lower technology (right). Diagrams are obtained by plotting equilibrium conditions (10) (for the canonical model), (12) (for the model with diminishing returns), (13) (for the model with wage rigidity), and (14) (for the model with job rationing) for a continuum of employment levels. The model with job rationing is calibrated in Table 1. Other models are calibrated in the online Appendix.

worker’s surplus, which is an increasing function of the wage. Hence, the wage falls, as is apparent in equation (9). Since it becomes cheaper to recruit a worker for any labor market tightness  $\theta$ , the marginal-recruiting-expense curve shifts down. The gross-marginal-profit curve is unchanged by construction. Therefore, equilibrium employment increases. For any employment  $n < 1$ , when recruiting cost converges

to 0 the marginal-recruiting-expense curve converges to 0 whereas the gross-marginal-profit curve remains constant and positive. As a consequence, firms hire workers until employment converges to 1.

### B. A Model with Diminishing Marginal Returns to Labor

ASSUMPTION 3: *There exists  $\alpha \in (0, 1)$  such that  $g(n, a) = a \cdot n^\alpha$ .*

ASSUMPTION 4: *There exists  $\beta \in (0, 1)$  such that*

$$(11) \quad w(n_t(i), \theta_t, n_t, a_t) = \frac{\alpha \cdot \beta \cdot a_t}{1 - \beta \cdot (1 - \alpha)} \cdot n_t(i)^{\alpha-1} + c \cdot (1 - s) \cdot \delta \cdot \beta \cdot \mathbb{E}_t[a_{t+1} \cdot \theta_{t+1}].$$

LEMMA 2: *Assume that the wage  $w_t(i)$  in any period  $t$  and firm  $i$  is determined by Stole and Zwiebel (1996) bargaining, and  $\beta \in (0, 1)$  is workers' bargaining power. Then  $w_t(i)$  satisfies (11).*

Lemma 2 shows that the wage schedule (11) is the Stole and Zwiebel (1996) bargaining solution, which allocates a fraction  $\beta$  of the marginal surplus to workers and the rest to the firm. The main departure from the canonical model is the introduction of diminishing marginal returns to labor ( $\alpha < 1$ ), which requires adapting the bargaining procedure as all workers have different marginal productivities. This model with diminishing marginal returns to labor retains the key elements of the models in Cahuc, Marque, and Wasmer (2008) and Elsby and Michaels (2008). In a static environment, the combination of firm's optimality condition (7) with wage (12) yields the following equilibrium condition:<sup>15</sup>

$$(12) \quad \left[ \frac{1 - \beta}{1 - \beta \cdot (1 - \alpha)} \right] \alpha \cdot n^{\alpha-1} = c \cdot \left[ \frac{1 - \delta \cdot (1 - s)}{q(\theta)} + \delta \cdot (1 - s) \cdot \beta \cdot \theta \right].$$

Equations (3) and (12) implicitly define equilibrium employment and labor market tightness as functions  $n(c)$  and  $\theta(c)$  of recruiting cost  $c$ , independent of  $a$ . As in the canonical model, unemployment does not fluctuate over the business cycle. Furthermore, Proposition 2 shows that despite diminishing returns to labor, the economy converges to full employment when matching frictions disappear.

PROPOSITION 2: *Under Assumption 3 and Assumption 4,  $\lim_{c \rightarrow 0} \theta(c) = +\infty$  and  $\lim_{c \rightarrow 0} n(c) = 1$ .*

<sup>15</sup>Equation (10) in the canonical model is a special case of equation (12), for  $\alpha = 1$ . This is because Nash bargaining is a special case of Stole and Zwiebel (1996) bargaining for a linear production function.

A diagram in Figure 1 explains the result. The upward-sloping line is marginal recruiting expenses, on the right-hand side of equation (12). The downward-sloping line is the marginal profit from hiring labor gross of recruiting expenses, on the left-hand side of equation (12). Here, the gross marginal profit is the marginal product of labor, minus the share of the wage independent of labor market tightness and recruiting cost, minus the marginal change in the wage bill implied by the renegotiation of the wage between the firm and its workers when an additional worker is hired. This curve is downward sloping by diminishing marginal returns to labor. When the recruiting cost decreases, the marginal-recruiting-expense curve shifts down while the gross-marginal-profit curve remains the same so employment increases, as in the canonical model. Since the gross marginal profit is always positive despite its negative slope—it intercepts the  $n = 1$  line at  $\alpha \cdot (1 - \beta) / [1 - \beta(1 - \alpha)] > 0$ —the economy converges asymptotically to full employment when recruiting cost falls to 0. Despite diminishing returns to labor, jobs are not rationed because bargained wages fall sufficiently when an increase in employment reduces the marginal product of labor.

C. A Model with Wage Rigidity

ASSUMPTION 5: *There exist  $\gamma \in [0, 1)$  and  $\omega \in (0, +\infty)$  such that*

$$w(n_t(i), \theta_t, n_t, a_t) = \omega \cdot a_t^\gamma.$$

I assume real wage rigidity in the form of the Blanchard and Galí (2010) wage schedule. The wage only partially adjusts to technology shocks because  $\gamma < 1$ . I also assume constant marginal returns to labor (Assumption 1). The model with wage rigidity retains the key elements of the models in Hall (2005a) and Blanchard and Galí (2010). If technology is bounded:  $a \in [\underline{a}, \bar{a}]$ , rigid wages are privately efficient if  $0 \leq \omega \leq \underline{a}^{1-\gamma}$  (Hall 2005a). In a static environment, firm’s optimality condition (7) becomes<sup>16</sup>

$$(13) \quad 1 - \omega \cdot a^{\gamma-1} = c \cdot \frac{1 - \delta \cdot (1 - s)}{q(\theta)}.$$

Equations (3) and (13) implicitly define equilibrium employment and labor market tightness as functions  $n(a, c)$  and  $\theta(a, c)$  of technology  $a$  and recruiting cost  $c$ .<sup>17</sup> When technology falls, the real wage does not fall as much as marginal productivity because it is rigid ( $\gamma < 1$ ), so unemployment increases. Hence, there are recessions in the sense that there are periods when unemployment is above average and above its socially efficient level, which is constant in this framework (Hosios 1990). Yet, as shown in Proposition 3, jobs are not rationed as the labor market clears at the limit without matching frictions.

<sup>16</sup>Equation (10) in the canonical model is a special case of equation (13), for  $\gamma = 1$  and  $\omega = (c\beta)/(1 - \beta) \cdot [1/q(\theta^*) + \delta(1 - s)(\theta^* - 1/q(\theta^*))]$  where  $\theta^*$  solves equation (10).

<sup>17</sup>Given the specification of the rigid wage schedule, equilibrium unemployment is unaffected by the absence of unemployment benefits or disutility of labor. Introducing benefits or disutility of labor does not affect theoretical and quantitative results for rigid-wage models satisfying Assumption 5.

**PROPOSITION 3:** *Suppose Assumption 1 and Assumption 5 hold, and that  $a \geq \omega^{1/(1-\gamma)}$ . Then  $\lim_{c \rightarrow 0} \theta(a, c) = +\infty$  and  $\lim_{c \rightarrow 0} n(a, c) = 1$ .*

A diagram in Figure 1 provides intuition. The upward-sloping line is marginal recruiting expenses, which is the amortized cost of recruiting, on the right-hand side of (13). The horizontal line is marginal profit gross of recruiting expenses, which is marginal product of labor minus wage, on the left-hand side of equation (13). This curve shifts when technology changes because of wage rigidity. When the recruiting cost decreases, the marginal-recruiting-expense curve shifts down whereas the gross-marginal-profit curve remains unchanged and positive because technology  $a \geq \omega^{1/(1-\gamma)}$ . Thus, equilibrium employment increases. As in the canonical model, firms hire workers until employment converges to 1 when recruiting cost converges to 0. If the wage is low enough for one match to be profitable, infinitely many matches would be profitable absent recruiting expenses and, in spite of wage rigidity, the economy would operate at full employment.

### III. A Model with Job Rationing

In existing models, the economy converges asymptotically to full employment in the absence of matching frictions. This section proposes a model in which jobs are rationed: in a world without matching frictions, wages remain above market-clearing level and some unemployment remains. Rationing arises naturally from the combination of diminishing marginal returns to labor (Assumption 3) and wage rigidity (Assumption 5).<sup>18</sup>

Any worker-firm match generates a positive bilateral surplus. To determine how the surplus is shared between worker and firm, I follow the reduced-form approach of the literature by assuming that wages adhere to the wage schedule from Assumption 5.<sup>19,20</sup> The rationality of a worker and a firm engaged in a long-term relationship imposes private efficiency: a positive share of the surplus must be allocated to each party; if not, the injured party would break the match and destroy the surplus. Lemma 3 establishes that wages are privately efficient if the wage schedule is flexible enough.

<sup>18</sup> Both assumptions have been used (but not combined) in the search-and-matching literature, and are standard in the broader macroeconomic literature. There is a long tradition of macroeconomic models whose short-run production function takes labor as the only variable input and has diminishing marginal returns to labor. Wage rigidity features in the many dynamic stochastic general equilibrium models that use staggered wage-setting mechanisms.

<sup>19</sup> Hall (2005a) shows that any privately efficient wage schedule is, each period, a Nash equilibrium of a simple wage-setting mechanism: the Nash (1953) demand game. But this mechanism does not resolve the indeterminacy of the outcome of wage setting, as any privately efficient wage is a possible Nash equilibrium. It does not capture the intricacies of wage setting in firms either. Thus, an important research agenda is to design a richer yet tractable wage-setting mechanism explaining the wage rigidity observed in the data, to improve our understanding of job rationing and unemployment fluctuations.

<sup>20</sup> The online Appendix shows that the properties of the model are virtually unchanged if the simple wage schedule  $w_t = \omega \cdot a_t^\gamma$  is replaced by an alternative wage schedule such that in period  $t$ , in jobs newly created at  $t$ ,  $w_{t,t} = \omega \cdot a_t^\gamma$ , and in existing jobs created at time  $\tau < t$ ,  $w_{\tau,t} = w_{\tau-1} \cdot (a_t/a_{\tau-1})^\zeta$ . This alternative wage schedule is more realistic as it allows for a wage flexibility  $\gamma$  in newly created jobs and  $\zeta$  in existing jobs. There is evidence that  $\gamma > \zeta$ : wages are more flexible in newly created jobs than in existing jobs (Pissarides 2009).

TABLE 1—CALIBRATION OF THE MODEL WITH JOB RATIONING (*Weekly Frequency*)

Interpretation	Value	Source
$\delta$ Discount factor	0.999	Corresponds to 5 percent annual rate
$s$ Separation rate	0.0095	JOLTS, 2000–2009
$\mu$ Efficacy of matching	0.233	JOLTS, 2000–2009
$\eta$ Unemployment-elasticity of matching	0.5	Petrongolo and Pissarides (2001)
$\gamma$ Real wage flexibility	0.7	Haefke, Sonntag, and Van Rens (2008)
$c$ Recruiting cost	0.215	$0.32 \times$ steady-state real wage
$\alpha$ Marginal returns to labor	0.666	Matches labor share of 0.66
$\omega$ Steady-state real wage	0.671	Matches unemployment rate of 5.8 percent
$\rho$ Autocorrelation of technology	0.992	MSPC, 1964–2009
$\sigma$ Standard deviation of shocks	0.0027	MSPC, 1964–2009

LEMMA 3: Let  $N(0, \sigma^2)$  be the centered, normal distribution with standard deviation  $\sigma$ . Suppose Assumption 3 and Assumption 5 hold, and that  $\ln(a_{t+1}) = \ln(a_t) + z_t$  with  $z_t \sim N(0, \sigma^2)$ . Then private efficiency condition (8) is violated with probability below  $p$  if

$$\gamma \geq 1 - (1 - \alpha) \cdot \frac{\ln(1 - s)}{\sigma \cdot \Phi^{-1}(p)},$$

where  $\Phi$  is the cumulative distribution function of  $N(0, 1)$ . In that case, condition (8) holds if technology shock  $z_t$  satisfies  $\Phi(z_t/\sigma) \geq p$ .

Section IV calibrates the wage schedule with  $\gamma = 0.7$  to match microestimates of wage rigidity, and calibrates the model from US data as shown in Table 1. It also constructs a technology series from US data. When I estimate detrended log technology as a random walk, I find that the conditional standard deviation is  $\sigma = 0.00265$  at weekly frequency. Under these calibrations, the wage schedule is flexible enough such that inefficient separations do not occur for negative technology shocks of amplitude below four standard deviations. Hence, inefficient separations never occur under normal circumstances.

The lower bound on the wage flexibility  $\gamma$  is independent of recruiting cost  $c$ , and is therefore valid in an environment without matching frictions. The condition is less stringent when the job destruction rate  $s$  is higher because job destructions reduce employment in firms, increasing the marginal product of labor at the beginning of each period through diminishing returns. For a given wage, a higher marginal product makes inefficient separations less likely. Through the same mechanism, a lower production-function parameter  $\alpha$  reduces the lower bound on  $\gamma$ .

#### A. Existence of Job Rationing

In a static environment, the firm's optimality condition (7) simplifies to

$$(14) \quad \alpha \cdot n^{\alpha-1} - \omega \cdot a^{\gamma-1} = [1 - (1 - s) \cdot \delta] \cdot \frac{c}{q(\theta)}.$$

The presence of recruiting cost  $c > 0$  creates a wedge between the marginal product of labor  $\alpha \cdot a \cdot n^{\alpha-1}$  and the wage  $\omega \cdot a^\gamma$  such that in equilibrium the marginal product of labor is strictly greater than the wage. Equations (3) and (14) uniquely define equilibrium employment and labor market tightness as implicit functions  $n(a, c)$  and  $\theta(a, c)$  of technology  $a$  and recruiting cost  $c$ . As in the model with wage rigidity in Section II, unemployment increases when technology falls because of wage rigidity. Thus, there are recessions in the sense that there are periods when unemployment is above average and above its socially efficient level. But this model goes one step further: it has the property that during these recessions, the labor market does not clear and some unemployment remains at the limit without matching frictions.

**PROPOSITION 4:** *Suppose Assumption 3 and Assumption 5 hold. If  $a \in (0, a^R)$  with  $a^R = (\omega/\alpha)^{1/(1-\gamma)}$ , there exists a unique  $n^R(a) \in (0, 1)$  such that  $\lim_{c \rightarrow 0} n(a, c) = n^R(a)$ ;  $n^R(a)$  is given by  $n^R(a) = (\alpha/\omega)^{1/(1-\alpha)} \cdot a^{(1-\gamma)/(1-\alpha)}$ . If  $a \geq a^R$ ,  $\lim_{c \rightarrow 0} n(a, c) = 1$ .*

When technology is low enough ( $a < a^R$ ), and without matching frictions ( $c = 0$ ), the firm's optimality condition (14) becomes

$$\alpha \cdot n^{\alpha-1} - \omega \cdot a^{\gamma-1} = 0.$$

It admits a unique solution  $n^R(a) < 1$ ;  $n^R(a)$  is equilibrium employment without matching frictions. In that case, jobs are rationed: the economy remains below full employment absent matching frictions. To measure the shortage of jobs on the labor market, I define rationing unemployment  $u^R(a)$ :

$$(15) \quad u^R(a) \equiv 1 - n^R(a) = 1 - \left(\frac{\alpha}{\omega}\right)^{\frac{1}{1-\alpha}} \cdot a^{\frac{1-\gamma}{1-\alpha}}.$$

Matching frictions impose recruiting expenses on firms, which contribute to the marginal cost of labor and lead firms to curtail employment further. To measure additional unemployment attributable to matching frictions, I define frictional unemployment  $u^F(a, c)$ :

$$(16) \quad u^F(a, c) \equiv u(a, c) - u^R(a).$$

If technology is high enough ( $a \geq a^R$ ), jobs are not rationed as the economy converges to full employment absent matching frictions. Then I define  $u^R(a) \equiv 0$  and  $u^F(a, c) \equiv u(a, c)$ : all unemployment is frictional.

The mechanism leading to job rationing is quite simple. Consider a large enough decline in technology. The prevailing wage does not fall as much as the marginal product of labor due to wage rigidity. In equilibrium, the wage is always below the marginal product of employed workers, but it is above the marginal product of the last workers in the labor force, who are less productive due to diminishing marginal returns to labor. Even without matching frictions, these workers cannot

be hired profitably and some unemployment remains.<sup>21</sup> In the absence of matching frictions, the labor market does not clear because there is no mechanism to bring wages down to market-clearing level. Firms cannot auction off jobs as in a perfectly competitive setting because they must wait to match with a job applicant to offer a wage. Then worker and firm share the bilateral surplus from their match, arising because the marginal product of labor always exceeds the flow value of unemployment. Nothing prevents the resulting wage from being above the marginal product of the least productive workers.

A diagram in Figure 1 depicts frictional and rationing unemployment. The upward-sloping line is marginal recruiting expenses, on the right-hand side of equation (14). The downward-sloping line is gross marginal profit, here the marginal product of labor minus the wage, on the left-hand side of equation (14). Rationing unemployment is unemployment prevailing when the recruiting cost  $c$  converges to zero. It is obtained at the intersection of the gross-marginal-profit curve with the  $x$ -axis because the marginal-recruiting-expense curve shifts down to the  $x$ -axis when  $c$  falls to 0 while the gross-marginal-profit curve remains unchanged. Total unemployment is obtained at the intersection of the gross-marginal-profit and marginal-recruiting-expense curves, and frictional unemployment is the difference between total and rationing unemployment. Diminishing returns to labor and wage rigidity are necessary for job rationing. Without diminishing returns, the gross-marginal-profit curve is horizontal and never intersects the  $x$ -axis on  $(0, 1)$ , so rationing unemployment is zero. Without wage rigidity, the gross-marginal-profit curve does not shift and could never intersect the  $x$ -axis, in which case rationing unemployment is zero. With rigid wages, the intersection occurs when technology is low enough.

### B. Cyclicalities of Frictional and Rationing Unemployment

In expansions, when technology is high enough ( $a \geq a^R$ ), matching frictions account for all unemployment. But when technology is low enough ( $a \leq a^R$ ), both job rationing and matching frictions contribute to unemployment. To understand how these two sources generate cyclical fluctuations in unemployment, I perform comparative statics with respect to technology that reveal how total, rationing, and frictional unemployments comove over the business cycle.

**PROPOSITION 5:** *Suppose Assumption 3 and Assumption 5 hold, and that  $a \in (0, a^R)$ . Then  $\partial u / \partial a < 0$ ,  $\partial u^R / \partial a < 0$ , and  $\partial u^F / \partial a > 0$ .*

The proposition establishes that around an equilibrium where jobs are rationed, when technology decreases, total unemployment increases, rationing unemployment increases, but paradoxically, frictional unemployment decreases. This proposition uncovers a novel mechanism behind unemployment fluctuations. When technology falls in recessions, the shortage of jobs becomes more acute, driving the rise in total unemployment. Simultaneously, the number of unemployed workers attributable to matching frictions falls because it becomes easier and cheaper for firms to recruit.

<sup>21</sup> Unlike with Nash bargaining or Stole and Zwiebel (1996) bargaining, wages are independent of recruiting cost. For a given tightness, wages do not fall when matching frictions decrease.

Figure 1 provides intuition. When technology decreases, the gross-marginal-profit curve shifts downward because the marginal product of labor falls while rigid wages adjust downward only partially. At the current employment level, gross marginal profit falls short of marginal recruiting expenses. Firms reduce hiring to increase gross marginal profit. Lower recruiting efforts by firms reduce labor market tightness and recruiting expenses. The adjustment process continues until a new equilibrium with higher unemployment is reached, when gross marginal profit equals marginal recruiting expenses. Rationing unemployment is mechanically higher. Irrespective of matching frictions, the shortage of jobs is more acute, and there are more unemployed workers and fewer vacancies. A firm posting a vacancy receives many applications from the large number of unemployed workers, and fills its vacancy rapidly at low cost. From the employment level prevailing when  $c = 0$ , a smaller reduction in employment suffices to bring the economy to equilibrium, so frictional unemployment is lower.

The fundamental property arising in a model with job rationing is that the shortage of jobs is more acute in recessions, while the amount of unemployment attributable to matching frictions is smaller. In other words, the reduction of unemployment that can be achieved by making recruiting costless for firms is smaller in recessions, even though total unemployment is higher. This property is unique to the model with job rationing. It does not hold in existing models in which, as explained in Section II, all unemployment is frictional. Clearly, this result has implications for macroeconomic policies concerned with search and matching on the labor market.<sup>22</sup> For the marginal analysis of policies, however, it is more relevant to determine the marginal reduction in unemployment that can be achieved by a marginal reduction in recruiting cost. This is what I study next.

### C. Cyclicalty of the Elasticity of Unemployment with Respect to Recruiting Cost

Proposition 6 characterizes the elasticity  $\epsilon_c^\theta$  and  $\epsilon_c^u$  of labor market tightness  $\theta$  and unemployment  $u$  with respect to recruiting cost  $c$  over the business cycle. It complements Proposition 5 by proving that, unlike in expansions, marginal variations in recruiting cost have little influence on labor market tightness and unemployment in recessions.

**PROPOSITION 6:** *Suppose Assumption 3 and Assumption 5 hold. Then*

$$\epsilon_c^\theta \equiv \frac{d\ln(\theta)}{d\ln(c)} = - \left[ \eta + (1 - \eta) \cdot (1 - \alpha) \cdot u \cdot \frac{\alpha \cdot q(\theta) \cdot n^{\alpha-1}}{c \cdot [1 - \delta \cdot (1 - s)]} \right]^{-1}$$

$$\epsilon_c^u \equiv \frac{d\ln(u)}{d\ln(c)} = -(1 - u) \cdot (1 - \eta) \cdot \epsilon_c^\theta.$$

<sup>22</sup> Search-and-matching models are widely used to analyze monetary policy (Blanchard and Galí 2010; Thomas 2008; Sala, Soderstrom, and Trigari 2008), unemployment insurance (Coles and Masters 2006; Lentz 2009), or active labor market policies (Holmlund and Linden 1993; Mortensen and Pissarides 1999).

Hence,  $\epsilon_c^\theta < 0$ ,  $\epsilon_c^u > 0$ ,  $d|\epsilon_c^\theta|/da > 0$ , and  $d|\epsilon_c^u|/da > 0$ .

An increase in recruiting cost reduces tightness and increases unemployment. But the effects of a marginal change in recruiting cost  $c$  on tightness  $\theta$  and unemployment  $u$ , measured by the elasticities  $|\epsilon_c^\theta|$  and  $|\epsilon_c^u|$ , are increasing with technology  $a$ . Thus, such a marginal change has little influence on tightness and unemployment in recessions.

The main force behind the drop in the amplitude of the elasticity  $|\epsilon_c^u|$  in recessions is the large increase of the job-filling probability  $q(\theta)$ . As it is easy for firms to recruit in recessions, recruiting expenses are only a small share of the marginal cost of labor, and fluctuations in recruiting cost barely change firm’s demand for labor. In fact, the elasticity  $\epsilon_c^u$  is related to the level of frictional unemployment. If there is some rationing ( $n^R < 1$ ) and unemployment is small enough ( $u \ll 1$ ), the elasticity is approximated by

$$\epsilon_c^u \approx \left[ \frac{\eta}{(1 - \eta)} + \frac{u}{u^F} \right]^{-1}.$$

In recessions, unemployment  $u$  increases and frictional unemployment  $u^F$  falls drastically as, in spite of matching frictions, it is easy and cheap for firms to recruit from the large pool of unemployed workers. Thus,  $u^F/u$ , and with it the elasticity  $\epsilon_c^u$ , decrease sharply.

This property is unique to the model with job rationing. In the model with wage rigidity, marginally reducing recruiting cost  $c$  has a positive effect  $\epsilon_c^\theta$  on labor market tightness  $\theta$  that remains constant even in recessions.<sup>23</sup> Corollary 1, which proves this result, complements Proposition 3, which showed that in the model with wage rigidity, reducing the recruiting cost to 0 would eliminate all unemployment even in recessions.

**COROLLARY 1:** *Suppose Assumption 1 and Assumption 5 hold, and that  $a \geq \omega^{1/(1-\gamma)}$ . Then  $\epsilon_c^\theta = -1/\eta$  and  $d\epsilon_c^\theta/da = 0$ .*

This result contrasts sharply with that of Proposition 6 for a model with job rationing. Figure 2 illustrates the contrast by comparing the elasticity  $\epsilon_c^\theta$  in the model with job rationing (with diminishing returns to labor,  $\alpha < 1$ ) to that in the model with wage rigidity (with constant returns to labor,  $\alpha = 1$ ). The elasticities are similar when unemployment is low. But in recessions, the elasticity remains constant in the model with wage rigidity, whereas it falls sharply in the model with job rationing.

Proposition 6 has critical macroeconomic implications. It suggests that reducing firm’s recruiting expenses, be it through higher search effort from unemployed workers, better matching efficiency, or weaker competition for workers from public jobs, has little effect on aggregate unemployment in recessions.

<sup>23</sup>In the canonical model and the model with diminishing marginal returns to labor, equilibrium conditions are independent from technology, and so are the elasticities  $\epsilon_c^u$  and  $\epsilon_c^\theta$ .

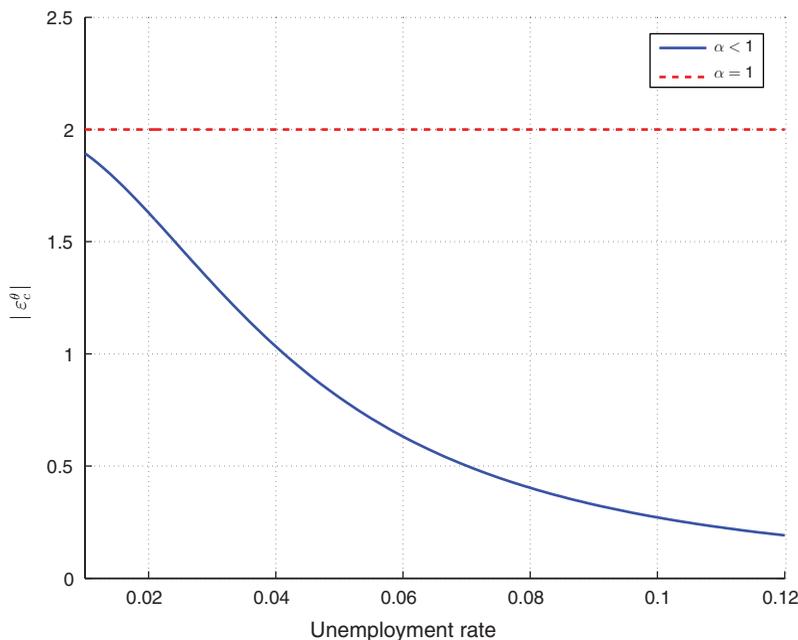


FIGURE 2. ELASTICITY OF LABOR MARKET TIGHTNESS WITH RESPECT TO RECRUITING COST

Notes: The elasticity in the model with job rationing ( $\alpha < 1$ ) is obtained from Proposition 6. For each unemployment level, labor market tightness is inferred using equation (3). The elasticity for the model with wage rigidity ( $\alpha = 1$ ) is given by Corollary 1. Calibration follows Table 1.

#### IV. Quantitative Analysis

This section calibrates wage rigidity and diminishing marginal returns to be consistent with micro-data on wage dynamics and macro-data on the labor share. I move beyond comparative statics by computing impulse response functions in the fully dynamic model. I also quantify comovements in total, frictional, and rationing unemployment when the model is simulated with actual US technology. A byproduct of these simulations is to verify that the calibrated model describes well the US labor market.

##### A. Calibration

I calibrate all parameters to weekly frequency, as summarized in Table 1.<sup>24</sup> I first calibrate the labor market parameters: job destruction rate ( $s$ ), recruiting cost ( $c$ ), and matching function ( $\mu, \eta$ ). I estimate the job destruction rate from the seasonally adjusted monthly series for total separations in all nonfarm industries

<sup>24</sup> A week is  $\frac{1}{4}$  of a month. Calibrating the model to weekly frequency is a good approximation to the continuous-time nature of unemployment flows. It keeps probabilities  $q(\theta)$  and  $f(\theta)$  in the  $(0, 1)$  interval without explicitly imposing such a constraint. Furthermore, an amount  $s \cdot n$  of beginning-of-period unemployment comes mechanically from the discrete inflow of labor into unemployment caused by job destructions at the end of each period. This amount, an artifact of the discrete-time structure of the model, explains the discrepancy between nonemployment  $1 - n$  and beginning-of-period unemployment  $u = 1 - (1 - s) \cdot n$ . This discrepancy is minimized by calibrating the model to weekly frequency to reduce the amount of job destruction per period. Models are commonly calibrated at such frequency (for example, Hagedorn and Manovskii 2008; Elsby and Michaels 2008).

constructed by the Bureau of Labor Statistics (BLS) from the Job Openings and Labor Turnover Survey (JOLTS) for the December 2000–June 2009 period.<sup>25</sup> The average separation rate is 0.038, so  $s = 0.0095$  at weekly frequency. Using micro-data gathered by Barron, Berger, and Black (1997), I estimate the per-period cost of opening a vacancy at 0.098 of a worker's wage.<sup>26</sup> This number accounts only for the labor cost of recruiting. Silva and Toledo (2009) include other expenses such as advertising costs, agency fees, and travel costs, to find that 0.42 of a worker's monthly wage is spent on each hire. Using the average monthly job-filling rate of 1.3 in JOLTS, 2000–2009, the flow cost of recruiting could be as high as 0.54 of a worker's wage. I calibrate recruiting cost as 0.32 of a worker's wage, the midpoint between the two previous estimates.<sup>27</sup> I specify the matching function as  $h(u, v) = \mu \cdot u^\eta \cdot v^{1-\eta}$  with  $\eta = 0.5$ , in line with empirical evidence (Petrongolo and Pissarides 2001). I estimate matching efficiency  $\mu$  with seasonally adjusted monthly series for number of hires and vacancies in all nonfarm industries, and for unemployment level, constructed by the BLS from JOLTS and the Current Population Survey (CPS) over the 2000–2009 period. For each month  $i$ , I calculate tightness  $\theta_i$  as the ratio of vacancy to unemployment and job-finding probability  $f_i$  as the ratio of hires to unemployment. The least squares estimate of  $\mu$ , which minimizes  $\sum_i (f_i - \mu \cdot \theta_i^{1-\eta})^2$ , is 0.93. At weekly frequency,  $\mu = 0.233$ .

Next I calibrate the wage flexibility  $\gamma$  based on estimates obtained in micro-data, less prone to composition effects than macro-data. The flexibility of wages in newly created jobs, and not that in existing jobs, matters for job creation.<sup>28</sup> Unfortunately, US micro-data are not rich enough to estimate wage flexibility in newly created jobs: these data follow workers over time but do not have information on the jobs workers take. The estimate closest to this flexibility in US data is provided by Haefke, Sonntag, and Van Rens (2008), who estimate the elasticity of total earnings of job movers with respect to productivity using panel data following a sample of production and supervisory workers over the 1984–2006 period.<sup>29</sup> They obtain an elasticity of 0.7, so I set  $\gamma = 0.7$ .<sup>30</sup>

<sup>25</sup> This is the longest period for which JOLTS is available. Earlier, comparable data were not available.

<sup>26</sup> Using the 1980 Employment Opportunity Pilot Project survey (2,994 observations), they find that employers spend on average 5.7 hours per offer, make 1.02 offers per hired worker, and take 13.4 days to fill a position. Hence the flow cost of maintaining a vacancy open is  $5.7/8 \times 1.02/13.4 \approx 0.054$  of a worker's wage. Adjusting for the possibility that hiring is done by supervisors who receive above-average wages as in Silva and Toledo (2009), the flow cost of keeping an open vacancy is 0.071 of a worker's wage. With the 1982 Employment Opportunity survey (1,270 observations), the flow cost is 0.106, and with the 1993 survey conducted for the W. E. Upjohn Foundation (210 observations), the flow cost is 0.117.

<sup>27</sup> Using the average unemployment rate and labor market tightness in JOLTS, I find that 0.9 percent of the total wage bill is spent on recruiting. The online Appendix discusses the sensitivity of the quantitative results to alternative calibrations of  $c$  in the range of possible values.

<sup>28</sup> I prove this result formally in online Appendix, using the fact that technology is very persistent and that only the expected present value of wages paid for the entire duration of a match matters for the firm's recruiting decision (Hall and Milgrom 2008). Pissarides (2009) proves a similar result in a different search-and-matching framework.

<sup>29</sup> If the composition of workers on each type of job remains the same over the business cycle, the flexibility of wages in newly created jobs is given by that of wages for newly hired workers. But the flexibility of wages for newly created jobs is different from that for newly hired workers if there is "cyclical upgrading," a change in the composition of jobs accepted by workers over the business cycle (Gertler and Trigari 2009). Firms could maintain rigid wages in each type of job, and workers taking new jobs could show procyclical wages if during expansions they face better opportunities to move to higher-paying industries, higher-paying firms within industry, or higher-paying jobs within firm.

<sup>30</sup> This elasticity is much higher than that of wages on existing jobs: the literature places the productivity-elasticity of wages of existing jobs in the 0.2–0.5 range for US data (Pissarides 2009).

So far, I have estimated parameters directly from micro- or macro-data. I now calibrate the steady-state wage  $\omega$  and the production function parameter  $\alpha$  such that the steady-state of the model matches the average US unemployment over the 1964–2009 period ( $\bar{u} = 5.8$  percent) and the conventional labor share ( $\bar{l}s = 0.66$ ). Unemployment is the seasonally adjusted monthly unemployment rate constructed by the BLS from the CPS. These targets imply steady-state employment  $\bar{n} = 0.951$  and steady-state labor market tightness  $\bar{\theta} = 0.446$ . In steady-state  $\bar{a} = 1$  and  $\bar{l}s \equiv (\bar{w} \cdot \bar{n})/\bar{y} = \omega \cdot \bar{n}^{1-\alpha}$ . Therefore equation (14) becomes  $\bar{l}s \cdot ([1 - \delta \cdot (1 - s)] \cdot 0.32/q(\bar{\theta}) + 1) = \alpha$ , yielding  $\alpha = 0.666$ ,  $\omega = 0.671$ , and  $c = 0.215$ .

### B. Simulated Moments

I verify that the model provides a sensible description of reality by comparing important simulated moments to their empirical counterparts in US data. I focus on second moments of the unemployment rate  $u$ , vacancy level  $v$ , labor market tightness  $\theta = v/u$ , real wage  $w$ , output  $y$ , and technology  $a$ . The sample period is 1964:I–2009:II. To construct a vacancy series for this period, I merge the vacancy data from JOLTS for 2001–2009, with the Conference Board help-wanted advertising index for 1964–2001.<sup>31</sup> I construct labor market tightness as the ratio of vacancy level to unemployment level. I take the quarterly average of these monthly series. Real wage is quarterly, average hourly earning in the nonfarm business sector constructed by the BLS Current Employment Statistics (CES) program, and deflated by the quarterly average of monthly Consumer Price Index (CPI) for all urban households, constructed by BLS. Output  $y$  and employment  $n$  are quarterly real output and employment in the nonfarm business sector constructed by the BLS Major Sector Productivity and Costs (MSPC) program. I construct log technology as a residual  $\ln(a) = \ln(y) - \alpha \cdot \ln(n)$ . All variables are seasonally adjusted, and to isolate fluctuations at business cycle frequency, I take the difference between log of the variables and a very low frequency trend—a Hodrick-Prescott (HP) filter with smoothing parameter  $10^5$ .<sup>32</sup> Table 2 presents these empirical moments.

Next, I estimate detrended log technology as an AR(1) process:  $\ln(a_{t+1}) = \rho \cdot \ln(a_t) + z_{t+1}$  with  $z_{t+1} \sim N(0, \sigma^2)$ . With quarterly data, I obtain an autocorrelation of 0.907 and a conditional standard deviation of 0.0089, which yields  $\rho = 0.992$  and  $\sigma = 0.00269$  at weekly frequency. I log-linearize the model around steady state and perturb it with i.i.d. technology shocks  $z_t \sim N(0, \sigma^2)$ .<sup>33</sup> I obtain weekly series of log-deviations for all variables. I discard the first 100 weeks of simulation. I record values every 12 weeks for quarterly series  $(y, w, a)$ . I record values every four weeks and take quarterly averages for monthly series  $(u, v, \theta)$ . I generate 182 quarters corresponding to data from 1964:I to 2009:II and detrend simulated data with an HP filter of smoothing parameter  $10^5$ . I repeat the simulation 100 times. Each sample

<sup>31</sup> The Conference Board index measures the number of help-wanted advertisements in major newspapers, and is a standard proxy for vacancies. The merger of both datasets is necessary since JOLTS begins only in December 2000 while the Conference Board data are less relevant after 2000, when many job openings became advertised on the Internet.

<sup>32</sup> The choice of a smoothing parameter of  $10^5$  is standard in the search-and-matching literature (for example, Shimer 2005; Mortensen and Nagypál 2007). In the online Appendix, I confirm that the quantitative results are robust to choosing a conventional smoothing parameter of 1,600.

<sup>33</sup> The online Appendix describes the log-linear model in details.

TABLE 2—SUMMARY STATISTICS, QUARTERLY US DATA, 1964–2009

	$u$	$v$	$\theta$	$w$	$y$	$a$
Standard deviation	0.170	0.184	0.342	0.021	0.030	0.020
Autocorrelation	0.919	0.940	0.930	0.955	0.898	0.881
Correlation	1	−0.889	−0.970	−0.234	−0.831	−0.561
	—	1	0.973	0.198	0.777	0.524
	—	—	1	0.223	0.824	0.559
	—	—	—	1	0.502	0.627
	—	—	—	—	1	0.891
	—	—	—	—	—	1

*Notes:* All data are seasonally adjusted. The sample period is 1964:I–2009:II. Unemployment rate  $u$  is quarterly average of monthly series constructed by the BLS from the CPS. Vacancy level  $v$  is quarterly average of monthly series constructed by merging data constructed by the BLS from the JOLTS and data from the Conference Board, as detailed in the text. Labor market tightness  $\theta$  is the ratio of vacancy level to unemployment level. Real wage  $w$  is quarterly, average hourly earning in the nonfarm business sector, constructed by the BLS CES program, and deflated by the quarterly average of monthly CPI for all urban households, constructed by BLS;  $y$  is quarterly real output in the nonfarm business sector constructed by the BLS MSPC program;  $\ln(a)$  is computed as the residual  $\ln(y) - \alpha \cdot \ln(n)$  where  $n$  is quarterly employment in the nonfarm business sector constructed by the BLS MSPC program. All variables are reported in log as deviations from an HP trend with smoothing parameter  $10^5$ .

provides an estimate of the means of model-generated data. I compute standard deviations of estimated means across samples to assess the precision of the predictions. Table 3 presents the results. Empirical and simulated moments for technology are similar as I calibrate the technology process to match the data. Other simulated moments are outcomes of the mechanics of the model.

The fit of the model is very good along several critical dimensions. First, the model amplifies technology shocks as much as observed in the data: in US data, a 1 percent decrease in technology increases unemployment by 4.2 percent, reduces vacancy by 4.3 percent, and reduces labor market tightness, measured by the vacancy-unemployment ratio, by 8.6 percent;<sup>34</sup> in the model, a 1 percent decrease in technology increases unemployment by 5.9 percent, reduces vacancy by 6.8 percent, and reduces labor market tightness by 12.7 percent. Second, wages are as rigid in the model as in the data: in both cases, a 1 percent decrease in technology only decreases wages by 0.7 percent. Third, simulated and empirical slopes of the Beveridge curve, measured by the correlation of unemployment with vacancy, are identical at  $-0.89$ .

The model does not perform well, however, along one important dimension: the simulated correlation of unemployment  $u$ , vacancy  $v$ , labor market tightness  $\theta$ , and wage  $w$  with technology  $a$  are close to 1, but empirical correlations are in the 0.5–0.65 range.<sup>35</sup> Hence, the simulated standard deviations of these variables are inferior

<sup>34</sup>The elasticity of unemployment with respect to technology  $\epsilon_a^u$  is the coefficient obtained in an ordinary least squares regression of log unemployment on log technology. This coefficient can be derived from Table 2:  $\epsilon_a^u = \rho(u, a) \times \sigma(u)/\sigma(a) = -0.478 \times 0.168/0.019 = -4.2$ . All other elasticities are computed similarly.

<sup>35</sup>There is evidence that wage rigidity induces a gradual change of wages in response to shocks (Bewley 1999). In the online Appendix, I explore the effect of a gradual response of wages of the form  $w_t = (\omega \cdot a_t^\zeta) \cdot (w_{t-1})^{1-\zeta}$  on the dynamics of the model;  $\zeta \in [0, 1]$  is the weight placed on the wage schedule targeted. In calibration, I choose  $\zeta = 0.016$  and keep  $\gamma = 0.7$ . Introducing gradual wage adjustment improves the fit of model by reducing the procyclicality of wages (the correlation of wage and technology is 0.67, in line with US data), without affecting the behavior of unemployment, vacancy, and labor market tightness (because the present value of wages paid to each worker for the duration of a worker-firm match does not change much, even though the timing of wage payments is different). But this improvement comes at a cost: it increases the rigidity of wages above what is observed in US data (the technology-elasticity of wages in the model (0.3) falls much below that in the data (0.7)).

TABLE 3—SIMULATED MOMENTS WITH TECHNOLOGY SHOCKS

	$u$	$v$	$\theta$	$w$	$y$	$a$
Standard deviation	0.091 (0.014)	0.113 (0.015)	0.199 (0.029)	0.011 (0.001)	0.019 (0.003)	0.015 (0.002)
Autocorrelation	0.898 (0.030)	0.762 (0.063)	0.858 (0.041)	0.812 (0.051)	0.834 (0.046)	0.812 (0.051)
Correlation	1	-0.891 (0.029)	-0.966 (0.010)	-0.979 (0.006)	-0.986 (0.004)	-0.979 (0.006)
	—	1	0.978 (0.005)	0.899 (0.027)	0.889 (0.030)	0.899 (0.027)
	—	—	1	0.961 (0.012)	0.959 (0.013)	0.961 (0.012)
	—	—	—	1	0.999 (0.001)	1.000 (0.000)
	—	—	—	—	1	0.999 (0.000)
	—	—	—	—	—	1

Notes: Results from simulating the log-linearized model with stochastic technology. All variables are reported in log as deviations from an HP trend with smoothing parameter  $10^5$ . Simulated standard errors (standard deviations across 100 simulations) are reported in parentheses.

to their empirical counterparts, even though amplification of technology shocks is realistic. The absence of demand or financial shocks from the model could explain these discrepancies.<sup>36</sup>

This simulation exercise contributes to our understanding of the role of wage rigidity in explaining unemployment fluctuations by showing that even a small amount of wage rigidity, such as that estimated by Haefke, Sonntag, and Van Rens (2008) using micro-data reporting earnings of new hires, is sufficient for the model to amplify shocks as much as in the data.<sup>37</sup> Furthermore, my results are consistent with those obtained by Martins, Solon, and Thomas (forthcoming) using matched employer-employee longitudinal data from Portugal, which track wages in newly created jobs. The authors show that their results are also relevant for the US labor market. They report a semielasticity of wages with respect to unemployment for newly created jobs of  $-1.8$ . In my model, the corresponding semielasticity in steady state is

$$e_u^w \equiv \frac{d\ln(w)}{du} = - \left[ \frac{1 - \gamma}{\gamma} \cdot \frac{1 - \eta}{\eta} \cdot \frac{\omega \cdot \bar{u} \cdot (1 - \bar{u})}{\alpha \cdot \bar{n}^{\alpha-1} \left[ 1 + (1 - \alpha) \cdot \frac{1 - \eta}{\eta} \cdot \bar{u} \right] - \omega} \right]^{-1}.$$

Using the calibration in Table 1,  $e_u^w = -1.6$ . Thus, the empirical semielasticity of wages for newly created jobs ( $-1.8$ ) is very close to that predicted theoretically

<sup>36</sup>Barnichon (2010b) shows that demand shocks are important to explain labor market fluctuations. Christiano, Trabandt, and Walentin (forthcoming) show that financial shocks drive a large share of labor market fluctuations, at least in small open economies. Future work should incorporate these disturbances into the model, and study how they affect the behavior of unemployment and its components.

<sup>37</sup>Numerous papers, starting with Hall (2005a) and Shimer (2004), showed that if wages in a search-and-matching model are sufficiently rigid, labor market tightness can be as volatile as in US data. This literature has not, however, calibrated wage rigidity using micro-evidence to examine whether the low amount of rigidity measured in wages for newly hired workers suffices to amplify technology shocks.

by our model (-1.6). As a consequence, the calibration of wage flexibility at  $\gamma = 0.7$  delivers realistic unemployment fluctuations, and realistic fluctuations in the expected present value of wages paid on new worker-firm matches.

### C. Impulse Response Functions

To confirm the comovements of technology with unemployment and its components in a stochastic environment, I compute impulse response functions (IRFs) in the log-linear model. In steady state,  $\bar{u} = 5.8$  percent,  $\bar{u}^R = 1 - (\alpha/\omega)^{1/(1-\alpha)} = 2.1$  percent, and  $\bar{u}^F = 3.7$  percent. Steady-state rationing unemployment is positive because steady-state technology  $\bar{a} = 1$  is below  $a^R = 1.024$ , which is the lowest technology level for which all unemployment is frictional. By contrast, in a static environment with  $a = a^R$ ,  $u = u^F = 4.8$  percent and  $u^R = 0$  percent. I extend the definitions of rationing and frictional unemployment given by equations (15) and (16) to a stochastic environment:

$$(17) \quad u_t^R \equiv \max\left\{0, 1 - \left(\frac{\alpha}{\omega}\right)^{\frac{1}{1-\alpha}} \cdot (a_{t-1})^{\frac{1-\gamma}{1-\alpha}}\right\}$$

$$(18) \quad u_t^F \equiv u_t - u_t^R.$$

Figure 3 shows the IRFs to a negative technology shock of one standard deviation ( $-\sigma = -0.00269$ ). On impact number of hires, labor market tightness, output, and wages fall discretely. In particular, labor market tightness drops by four percent in response to a drop in technology by 0.27 percent. The implied elasticity of labor market tightness with respect to technology is  $4/0.27 = 14.8$ , consistent with the simulated elasticity of 12.7 computed above from Table 3. The reduced number of hirings together with the constant amount of job destruction lead unemployment to build up and peak around four months after the technology shock, in line with the findings in Stock and Watson (1999).

The IRFs uncover the dynamic behavior of unemployment and its component. Technology falls on impact in period  $t$ . The shortage of jobs is immediately more acute, as wages are rigid: rationing unemployment jumps up. This acute shortage of jobs drives the rise in unemployment. Unemployment would jump up like rationing unemployment if firms completely stopped recruiting in period  $t$ .<sup>38</sup> The unemployment rise is much slower than if firms stopped recruiting because firms keep on recruiting after a negative shock, albeit at a slower pace. As firms have the ability to substitute recruiting intertemporally, they find it optimal to do so to reduce recruiting expenses.<sup>39</sup> The optimal response to the presence of matching frictions

<sup>38</sup> Employment could decrease at a weekly rate of about one percent since the job separation rate is  $s = 0.0095$ . So unemployment could increase at a weekly rate of 15 percent since  $d\ln(u) = (1 - \bar{u})/\bar{u} \cdot d\ln(n)$ . The log-deviation of unemployment depicted in the IRF could gain 0.15 each week, much more than the observed increase of 0.016 over 4 months.

<sup>39</sup> Assume that firms stop recruiting at time  $t$  to reduce employment as new matches are less profitable, and that the labor market is again in steady state at a higher unemployment level with balanced flows into and out of unemployment in period  $t + 1$ . Firms face a tightness  $\theta_t = 0$  at time  $t$  as firms stop recruiting, and a much higher

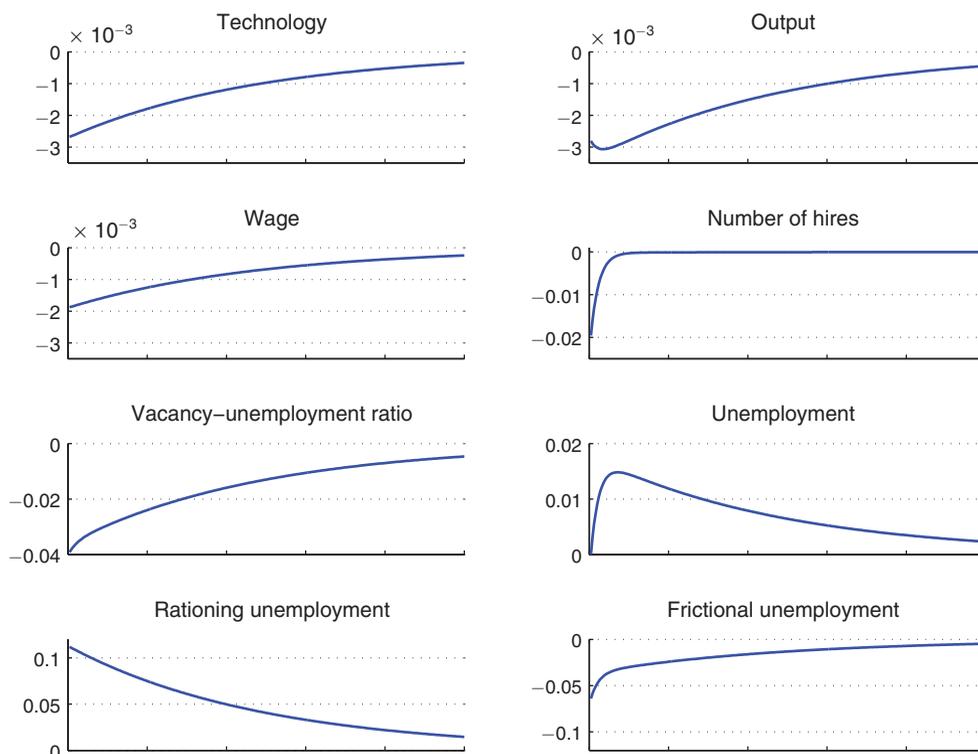


FIGURE 3. IMPULSE RESPONSE FUNCTIONS TO A NEGATIVE TECHNOLOGY SHOCK

Notes: Impulse response functions (IRFs) represent the logarithmic deviation from steady state for each variable. IRFs are obtained by imposing a negative technology shock of  $-\sigma = -0.00269$  to the log-linear model. The time period displayed on the  $x$ -axis is 250 weeks.

is to substitute recruiting from the future to the present to take advantage of the currently slack labor market. After an adverse shock, matching frictions slow down the growth of unemployment, and the amplitude of the drop in frictional unemployment on impact captures the amount of unemployment avoided by intertemporal substitution of recruiting. As technology remains below steady state, the shortage of jobs remains acute and rationing unemployment remains above steady state. Unemployment increases as firms reduce hiring. It becomes easier to recruit workers. Frictional unemployment is lower by the static effect of Proposition 5, thus mitigating the increase in rationing unemployment. These static effects, combined with the dynamic effects highlighted above, generate cyclical fluctuations in unemployment and its components.

#### D. Historical Decomposition of Unemployment

To assess how much unemployment and its components fluctuate over the cycle, I construct historical time series for rationing and frictional unemployment from the

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tightness  $\theta_{t+1} \gg 0$  the next period given by the Beveridge curve (3). In that case, firms increase profits by substituting recruiting from period  $t + 1$  to period  $t$ , as recruiting is free at  $t$  but very costly at  $t + 1$ .

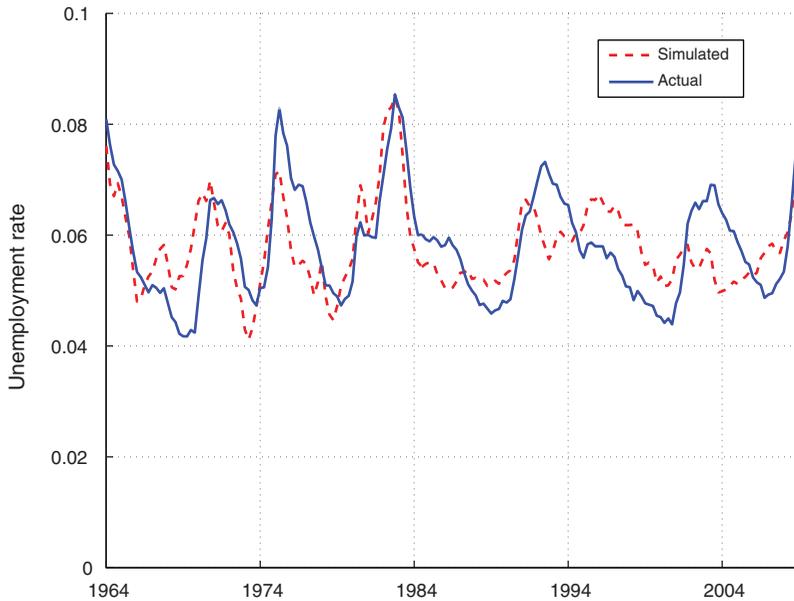


FIGURE 4. ACTUAL AND SIMULATED US UNEMPLOYMENT, 1964–2009

*Notes:* Actual unemployment rate is quarterly average of the monthly, seasonally adjusted series constructed by the BLS from the CPS. Simulated unemployment rate is generated when the nonlinear model is stimulated by the quarterly technology series constructed in the text, and solved with the Fair and Taylor (1983) shooting algorithm. I detrend log of technology, actual unemployment, and simulated unemployment with an HP filter with smoothing parameter  $10^5$ . The time period is 1964:I–2009:II.

technology series measured in US data.<sup>40</sup> In this simulation, the economy departs substantially from the steady state, so I do not use the log-linear model. Instead I solve exactly the nonlinear model with the Fair and Taylor (1983) shooting algorithm.<sup>41</sup> Figure 4 shows that model-generated and actual unemployment match well. Both series have comparable standard deviations: 0.010 for actual series and 0.009 for simulated series. The match is remarkably good given the simplicity of the model: the correlation of the two series is 0.65. The results are even better on the first half of the sample (91 quarters from 1964:I–1986:III). Standard deviations are 0.011 for actual series and 0.010 for simulated series. The correlation between the two series is 0.73. Thus, the model explains a good amount of fluctuations in

<sup>40</sup>Fluctuations in technology measured as a residual  $\ln(a) = \ln(y) - \alpha \ln(n)$  could be partly endogenous. The online Appendix alleviates this concern by showing that the technology series measured as a residual in US data is in fact very similar to the utilization-adjusted total factor productivity (TFP) series from Fernald (2009), which controls for labor hoarding and variable capital utilization. Accordingly, quantitative results are robust to using the utilization-adjusted TFP series instead of the measured technology series.

<sup>41</sup>The algorithm solves dynamic rational expectation models. Each period, it iterates over the path of expected values for endogenous (employment and labor market tightness) and exogenous (technology) variables, until convergence to a path of rational expectations consistent with the predictions of the model. Since the model operates at weekly frequency, I interpolate the detrended quarterly technology series into a weekly series, with which I simulate the model. To simplify computation of expectations, I discretize the AR(1) process for detrended technology as a 200-state Markov chain (Tauchen 1986).

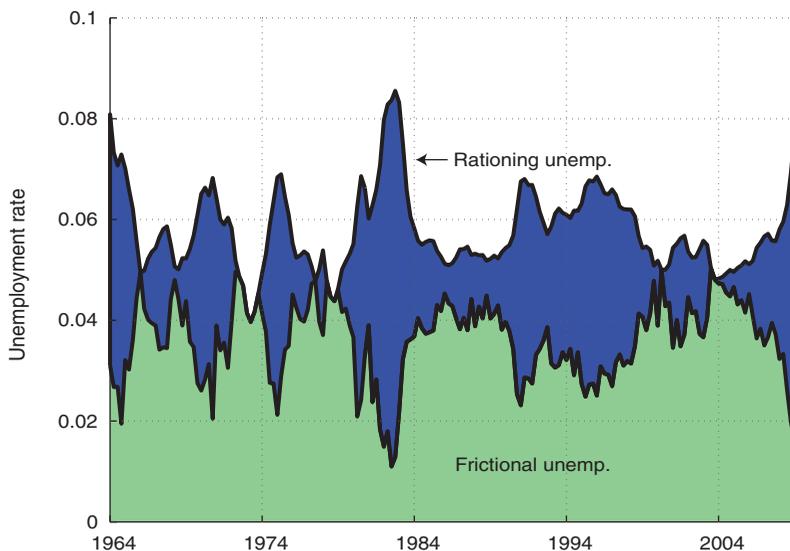


FIGURE 5. DECOMPOSITION OF SIMULATED US UNEMPLOYMENT, 1964–2009

*Notes:* The graph decomposes the unemployment series generated when the nonlinear model is stimulated by the detrended quarterly technology series constructed in the text. The period is 1964:I–2009:II. I solve the model with the Fair and Taylor (1983) shooting algorithm. Frictional and rationing unemployment are constructed from equations (17) and (18).

unemployment, and captures all seven US recessions in the 1964–2009 period, but for the 2001 recession.<sup>42,43</sup>

From technology measured in US data and simulated unemployment, I generate rationing and frictional unemployment using equations (17) and (18). Figure 5 shows the resulting decomposition of simulated unemployment. When simulated unemployment is below 5.0 percent, it is solely frictional. Above 5.0 percent, both rationing and frictional unemployment contribute to unemployment. Spikes in simulated unemployment are driven by steep rises in rationing unemployment, and accompanied by sharp drops in frictional unemployment, as illustrated by current events. In 2004:I, simulated unemployment was at 4.8 percent, all of which was frictional. When simulated unemployment peaked at 8.1 percent in 2009:II, frictional unemployment dropped to 1.8 percent and rationing unemployment climbed to 6.3 percent.<sup>44</sup>

<sup>42</sup> According to the NBER business cycle dating committee, the seven recessions during this period are: 1969–1970, 1973–1975, 1980, 1981–1982, 1990–1991, 2001, and 2007–2009.

<sup>43</sup> Alternatively, the online Appendix determines the technology series such that model-generated unemployment exactly matches actual unemployment, and construct rationing and frictional unemployment rates from these series. In that case, rationing and frictional unemployment add up to observed unemployment. This decomposition confirms the quantitative findings. I also introduce additional shocks (to the wage schedule and to the matching function), to match additional observable variables (output and labor market tightness) and confirm the robustness of the quantitative results.

<sup>44</sup> Private efficiency is satisfied during the sample period as hiring always remains positive.

## V. Summary and Normative Implications

This paper integrates matching frictions and job rationing to offer a treatment of unemployment that accounts for labor market flows, costly recruiting, and a possible shortage of jobs resulting from a failure of the labor market to clear in the absence of matching frictions. The fundamental property of the model is that, although the labor market always sees vast flows of jobs and workers and a great deal of matching activity, recessions are periods of acute job shortage during which matching frictions do not matter. The model departs from the literature because in existing search-and-matching models, even those with rigid wages, jobs are never rationed: the economy converges to full employment as matching frictions become arbitrarily small.

The model with job rationing has a wide range of implications for the design of macroeconomic policies concerned with search and matching on the labor market, such as unemployment insurance (UI), placement agency, or public employment. For instance, Landais, Michailat, and Saez (2010) build on the model to characterize optimal UI over the business cycle. In recessions, aggregate job-search efforts have little influence on aggregate unemployment. While unemployment benefits do reduce job-search efforts in recessions, this reduction only increases unemployment negligibly. Therefore, the cost of UI from higher unemployment through reduced search efforts decreases, whereas the insurance value of UI from consumption smoothing remains constant. Hence, they prove that optimal UI is more generous in recessions than in expansions. Building on the model as well, Michailat (2011) shows that placement agencies, which monitor workers' search efforts to improve matching on the labor market, are only effective in expansions when all unemployment is frictional. In recessions, matching frictions do not matter much and placement agencies are ineffective. Michailat (2011) also studies direct employment programs, which hire unemployed workers in public-sector jobs. Public jobs crowd out private jobs because these jobs compete to hire the same unemployed workers. The effectiveness of direct employment depends on how much it crowds out private employment. When unemployment is high, crowding out is low because matching frictions are unimportant, so competition across firms to match with workers is not a hindrance on job creation. Hence, direct employment is effective in recessions. Landais, Michailat, and Saez (2010) and Michailat (2011) also prove that these results do not hold in existing search-and-matching models, even those with wage rigidity. In existing models with wage rigidity, at any point of the business cycle, optimal unemployment insurance remains low, placement agencies remain effective, and public jobs crowd out private jobs one-for-one.

## REFERENCES

- Abraham, Katharine G., and Lawrence F. Katz. 1986. "Cyclical Unemployment: Sectoral Shifts or Aggregate Disturbances?" *Journal of Political Economy* 94 (3): 507–22.
- Akerlof, George A. 1980. "A Theory of Social Custom, of Which Unemployment May be One Consequence." *Quarterly Journal of Economics* 94 (4): 749–75.
- Akerlof, George A. 1982. "Labor Contracts as Partial Gift Exchange." *Quarterly Journal of Economics* 97 (4): 543–69.

- Alexopoulos, Michelle.** 2004. "Unemployment and the Business Cycle." *Journal of Monetary Economics* 51 (2): 277–98.
- Alexopoulos, Michelle.** 2007. "A Monetary Business Cycle Model with Unemployment." *Journal of Economic Dynamics and Control* 31 (12): 3904–40.
- Baker, George, Michael Gibbs, and Bengt Holmstrom.** 1994. "The Wage Policy of a Firm." *Quarterly Journal of Economics* 109 (4): 921–55.
- Barnichon, Régis.** 2010a. "The Lumpy Job Separation Rate: Reconciling Search Models with the Ins and Outs of Unemployment." <http://sites.google.com/site/regisbarnichon/cv/Lumpyjs.pdf>.
- Barnichon, Régis.** 2010b. "Productivity and Unemployment over the Business Cycle." *Journal of Monetary Economics* 57 (8): 1013–25.
- Barro, Robert J.** 1977. "Long-Term Contracting, Sticky Prices, and Monetary Policy." *Journal of Monetary Economics* 3 (3): 305–16.
- Barron, John M., Mark C. Berger, and Dan A. Black.** 1997. "Employer Search, Training, and Vacancy Duration." *Economic Inquiry* 35 (1): 167–92.
- Bewley, Truman F.** 1999. *Why Wages Don't Fall During a Recession*. Cambridge, MA: Harvard University Press.
- Blanchard, Olivier, and Peter Diamond.** 1989. "The Beveridge Curve." *Brookings Papers on Economic Activity* (1): 1–60.
- Blanchard, Olivier, and Jordi Galí.** 2010. "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment." *American Economic Journal: Macroeconomics* 2 (2): 1–30.
- Burdett, Kenneth, and Dale T. Mortensen.** 1998. "Wage Differentials, Employer Size, and Unemployment." *International Economic Review* 39 (2): 257–73.
- Cahuc, Pierre, Francois Marque, and Etienne Wasmer.** 2008. "A Theory of Wages and Labor Demand with Intra-firm Bargaining and Matching Frictions." *International Economic Review* 49 (3): 943–72.
- Campbell, Carl M., III, and Kunal S. Kamalani.** 1997. "The Reasons for Wage Rigidity: Evidence from a Survey of Firms." *Quarterly Journal of Economics* 112 (3): 759–89.
- Carlsson, Mikael, Stefan Eriksson, and Nils Gottfries.** 2006. "Testing Theories of Job Creation: Does Supply Create Its Own Demand?" Institute for the Study of Labor (IZA) Discussion Paper 2024.
- Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin.** Forthcoming. "Introducing Financial Frictions and Unemployment into a Small Open Economy Model." *Journal of Economic Dynamics and Control*.
- Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin.** 2010. "Involuntary Unemployment and the Business Cycle." National Bureau of Economic Research Working Paper 15801.
- Coles, Melvyn, and Adrian Masters.** 2006. "Optimal Unemployment Insurance in a Matching Equilibrium." *Journal of Labor Economics* 24 (1): 109–38.
- Davis, Steven J., John C. Haltiwanger, and Scott Schuh.** 1996. *Job Creation and Destruction*. Cambridge, MA: MIT Press.
- Doeringer, Peter B., and Michael J. Piore.** 1971. *Internal Labor Markets and Manpower Analysis*. Lexington, MA: Heath, Lexington Books.
- Elsby, Michael W. L., and Ryan Michaels.** 2008. "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows." National Bureau of Economic Research Working Paper 13777.
- Elsby, Michael W. L., Ryan Michaels, and Gary Solon.** 2009. "The Ins and Outs of Cyclical Unemployment." *American Economic Journal: Macroeconomics* 1 (1): 84–110.
- Fair, Ray C., and John B. Taylor.** 1983. "Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models." *Econometrica* 51 (4): 1169–85.
- Fernald, John G.** 2009. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity." <http://www.frbfsf.org/economics/economists/jfernal/Quarterly-TFP.pdf>.
- Fujita, Shigeru, and Garey Ramey.** 2009. "The Cyclicalities of Separation and Job Finding Rates." *International Economic Review* 50 (2): 415–30.
- Galí, Jordi.** 2011. *Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective*. Cambridge, MA: MIT Press.
- Galí, Jordi, Frank Smets, and Rafael Wouters.** Forthcoming. "Unemployment in an Estimated New Keynesian Model." In *NBER Macroeconomics Annual 2011*, edited by Daron Acemoglu and Michael Woodford. Chicago: University of Chicago Press.
- Gertler, Mark, and Antonella Trigari.** 2009. "Unemployment Fluctuations with Staggered Nash Wage Bargaining." *Journal of Political Economy* 117 (1): 38–86.
- Gertler, Mark, Luca Sala, and Antonella Trigari.** 2008. "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining." *Journal of Money, Credit, and Banking* 40 (8): 1713–64.

- Gorodnichenko, Yuriy, Enrique G. Mendoza, and Linda L. Tesar. 2012. "The Finnish Great Depression: From Russia with Love." *American Economic Review* 102 (4): TK.
- Haefke, Christian, Marcus Sonntag, and Thijs van Rens. 2007. "Wage Rigidity and Job Creation." Institute for the Study of Labor (IZA) Discussion Paper 3714.
- Hagedorn, Marcus, and Iourii Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review* 98 (4): 1692–1706.
- Hall, Robert E. 2005a. "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review* 95 (1): 50–65.
- Hall, Robert E. 2005b. "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market." *Review of Economics and Statistics* 87 (3): 397–407.
- Hall, Robert E., and Paul R. Milgrom. 2008. "The Limited Influence of Unemployment on the Wage Bargain." *American Economic Review* 98 (4): 1653–74.
- Holmlund, Bertil, and Johan Linden. 1993. "Job Matching, Temporary Public Employment, and Equilibrium Unemployment." *Journal of Public Economics* 51 (3): 329–43.
- Hosios, Arthur J. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies* 57 (2): 279–98.
- Jacoby, Sanford M. 1984. "The Development of Internal Labor Markets in American Manufacturing Firms." In *Internal Labor Markets*, edited by P. Osterman, 23–69. Cambridge, MA: MIT Press.
- Krueger, Alan B., and Alexandre Mas. 2004. "Strikes, Scabs, and Tread Separations: Labor Strife and the Production of Defective Bridgestone/Firestone Tires." *Journal of Political Economy* 112 (2): 253–89.
- Landais, Camille, Pascal Michailat, and Emmanuel Saez. 2010. "Optimal Unemployment Insurance over the Business Cycle." National Bureau of Economic Research Working Paper 16526.
- Lentz, Rasmus. 2009. "Optimal Unemployment Insurance in an Estimated Job Search Model with Savings." *Review of Economic Dynamics* 12 (1): 37–57.
- Lindbeck, Assar, and Dennis J. Snower. 1988. *The Insider-Outsider Theory of Employment and Unemployment*. Cambridge, MA: MIT Press.
- Martins, Pedro S., Gary Solon, and Jonathan P. Thomas. Forthcoming. "Measuring What Employers Really Do about Entry Wages over the Business Cycle: A New Approach." *American Economic Journal: Macroeconomics*.
- Mas, Alexandre. 2006. "Pay, Reference Points, and Police Performance." *Quarterly Journal of Economics* 121 (3): 783–821.
- Michailat, Pascal. 2011. "Fiscal Multipliers over the Business Cycle." CEPR Discussion Paper 8610.
- Michailat, Pascal. 2012. "Do Matching Frictions Explain Unemployment? Not in Bad Times: Dataset." *American Economic Review*. <http://dx.doi.org/10.1257/aer.102.4.1721>.
- Mortensen, Dale T., and Éva Nagypál. 2007. "More on Unemployment and Vacancy Fluctuations." *Review of Economic Dynamics* 10 (3): 327–47.
- Mortensen, Dale T., and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies* 61 (3): 397–415.
- Mortensen, Dale T., and Christopher A. Pissarides. 1999. "New Developments in Models of Search in the Labor Market." In *Handbook of Labor Economics Volume 3B*, edited by O. Ashenfelter and D. Card, 2567–2627. Amsterdam: Elsevier Science, North-Holland.
- Nash, J. F. 1953. "Two-Person Cooperative Games." *Econometrica* 21 (1): 128–40.
- Petrongolo, Barbara, and Christopher A. Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature* 39 (2): 390–431.
- Pissarides, Christopher A. 1985. "Short-Run Equilibrium Dynamics of Unemployment Vacancies, and Real Wages." *American Economic Review* 75 (4): 676–90.
- Pissarides, Christopher A. 2000. *Equilibrium Unemployment Theory*. 2nd ed. Cambridge, MA: MIT Press.
- Pissarides, Christopher A. 2009. "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica* 77 (5): 1339–69.
- Sala, Luca, Ulf Soderstrom, and Antonella Trigari. 2008. "Monetary Policy under Uncertainty in an Estimated Model with Labor Market Frictions." *Journal of Monetary Economics* 55 (5): 983–1006.
- Shimer, Robert. 2004. "The Consequences of Rigid Wages in Search Models." *Journal of the European Economic Association* 2 (2–3): 469–79.
- Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review* 95 (1): 25–49.
- Shimer, Robert. 2007. "Mismatch." *American Economic Review* 97 (4): 1074–1101.
- Shimer, Robert. 2012. "Reassessing the Ins and Outs of Unemployment." *Review of Economic Dynamics* 15(2): 127–48.
- Shimer, Robert. 2010. *Labor Markets and Business Cycles*. Princeton, NJ: Princeton University Press.

- Silva, José Ignacio, and Manuel Toledo.** 2009. "Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment." *Macroeconomic Dynamics* 13 (S1): 76–96.
- Solow, Robert M.** 1979. "Another Possible Source of Wage Stickiness." *Journal of Macroeconomics* 1 (1): 79–82.
- Stock, James H., and Mark W. Watson.** 1999. "Business Cycle Fluctuations in U.S. Macroeconomic Time Series." In *Handbook of Macroeconomics. Vol. 1*, edited by John B. Taylor and Michael Woodford, 3–64. Amsterdam: Elsevier.
- Stole, Lars A., and Jeffrey Zwiebel.** 1996. "Intra-firm Bargaining under Non-binding Contracts." *Review of Economic Studies* 63 (3): 375–410.
- Tauchen, George.** 1986. "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions." *Economic Letters* 20 (2): 177–181.
- Temin, Peter.** 1990. "Socialism and Wages in the Recovery from the Great Depression in the United States and Germany." *Journal of Economic History* 50 (2): 297–307.
- Thomas, Carlos.** 2008. "Search and Matching Frictions and Optimal Monetary Policy." *Journal of Monetary Economics* 55 (5): 936–56.
- Yashiv, Eran.** 2000. "The Determinants of Equilibrium Unemployment." *American Economic Review* 90 (5): 1297–1322.